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LF AND MF PROPAGATION:
The effect of earth curvature on sea gain

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Summary

If an l.f. or m.f. aerial is situated near the sea, the strength of sky-wave signals is increased by sea gain. The report shows that Earth curvature has a significant effect at l.f. and m.f. on sea gain at coastal sites but has less influence on the reduction of sea gain which occurs when an aerial moves inland or when the sea is of limited extent.

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LF AND MF PROPAGATION:
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1. Introduction

The strength of the low-angle sky-wave produced by an l.f. or m.f. transmitter is greatest if it radiates over open sea from a coastal site. The increase in signal strength, compared with that due to a transmitter well inland, is known as sea gain. The sea gain is reduced if the transmitter is some distance inland or if the sea is of very limited extent. Sea gain also occurs at the receiving end of the path, where field strengths produced by downcoming sky waves are highest near the sea.

The report considers the effect of Earth curvature on sea gain. Attention is confined to vertical polarisation because conventional l.f. and m.f. transmitting aerials radiate vertically polarised waves, while domestic receiving aerials respond mainly to vertical polarisation. Section 2 considers the sea gain which occurs if land of unlimited extent is replaced by sea water; this is the sea gain which applies to an aerial situated on a sea coast which is unobstructed by further land. Section 3 considers the effect of a land/sea boundary; the theory described in Section 3 enables sea gain to be calculated either for an aerial some distance inland or for an aerial situated on the edge of a narrow sea inlet or channel.

2. Sea gain for a homogenous Earth

If the Earth were perfectly flat and of uniform conductivity, the normalised vertical radiation pattern (v.r.p.) of a short vertical aerial would be given by:

$$A = \frac{1 + R_v(\psi)}{2} \cos \psi \qquad (1)$$

where $R_v(\psi)$ = Fresnel reflection coefficient for vertically-polarised plane waves
ψ = angle of elevation of sky wave.

The effect of Earth curvature has been studied by Wait and Conda,[1] who have shown that Equation (1) is reasonably accurate for values of ψ greater than about 3°. At lower angles, however, the field strength exceeds the value given by Equation (1) because waves diffract around

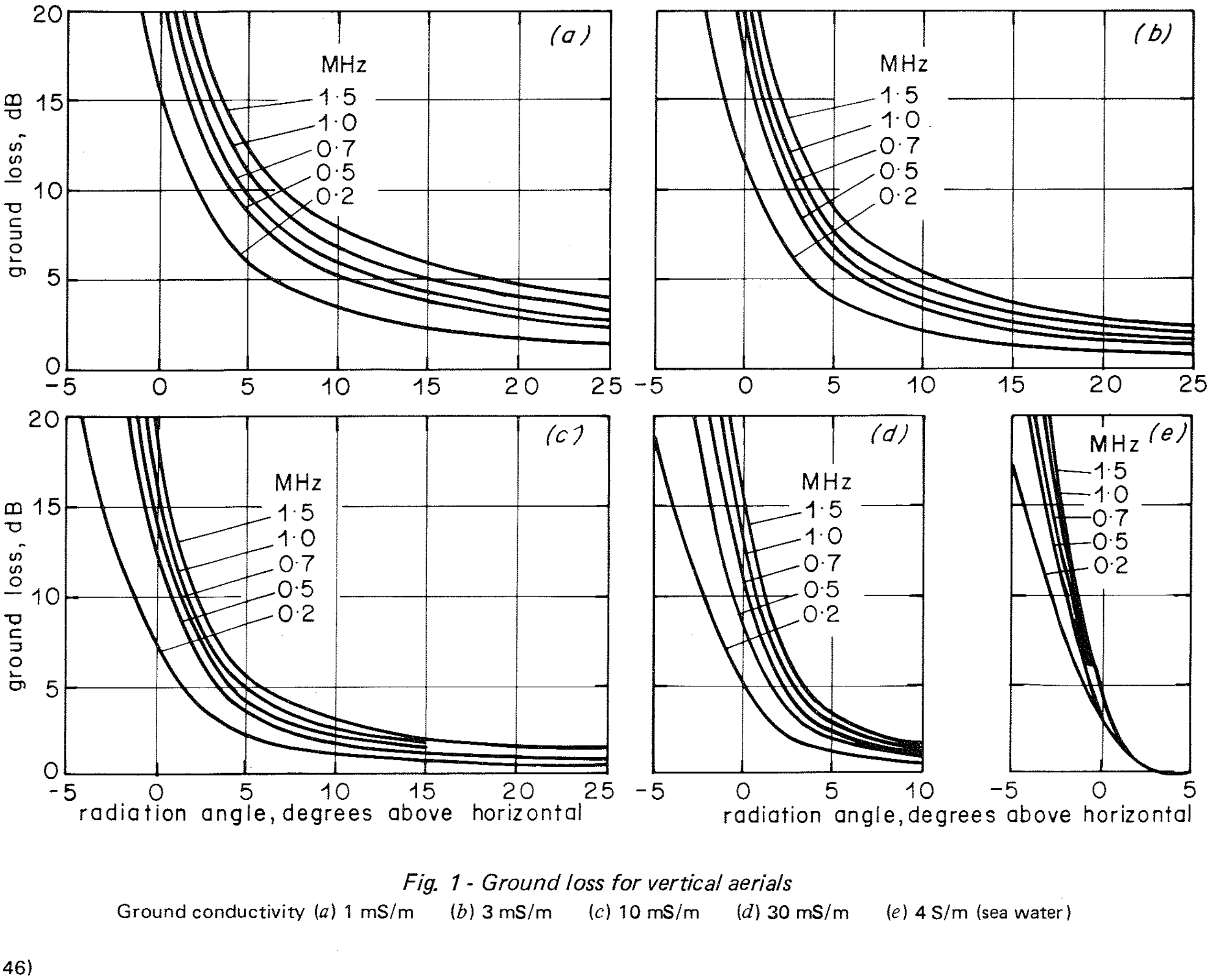

Fig. 1 - Ground loss for vertical aerials
Ground conductivity (*a*) 1 mS/m (*b*) 3 mS/m (*c*) 10 mS/m (*d*) 30 mS/m (*e*) 4 S/m (sea water)

the curvature of the Earth before propagating towards the ionosphere.

In calculating the combined effect of diffraction and imperfect ground conductivity on the strength of radiated signals, a short vertical aerial over flat perfectly-conducting ground provides a convenient reference. The reduction in field strength which occurs when the flat perfectly-conducting ground is replaced by an imperfectly-conducting curved Earth may then be calculated; this reduction is known as ground loss.

Fig. 1 shows ground loss curves, calculated by the BBC from Wait and Conda's theory, which have been published by the CCIR.[2] Atmospheric refraction was taken into account by multiplying the Earth's radius by factors derived from Reference 3. Fig. 1 extends to negative radiation angles, which apply when waves diffract around the curvature of the Earth and which are defined as the angular distance between the aerial and the tangent point for the wave. Although these curves were calculated for short vertical aerials they may be used for aerials up to 0·6λ high with negligible error and apply equally to transmitting and receiving aerials.

Fig. 1 shows that ground loss for sea water is less than that for land. The difference in decibels between the two losses is known as sea gain; this is the increase in signal strength which occurs when land is replaced by sea water. Comparison of sea gains calculated for flat Earth from Equation (1) and for curved Earth from Fig. 1 shows that Earth curvature reduces sea gain by several decibels at low radiation angles.

3. Sea gain in the presence of a land/sea boundary

In Section 2 the land or sea was assumed to be of infinite extent. Sea gain is, however, modified by the presence of land/sea boundaries. It has been shown experimentally that sea gain is reduced if a receiver is moved inland from the coast[4] and sea gain will also be reduced if the sea in the vicinity of an aerial is of limited extent.

The effect of a land/sea boundary on the v.r.p. of a short vertical aerial has been studied theoretically by Andersen, assuming the Earth to be flat.[5] Andersen was concerned with propagation at 10 MHz and Earth curvature has no effect on sea gain at that frequency. Earth curvature has been taken into account by Wait,[6] who has obtained a theoretical solution for the v.r.p. of a small loop aerial near a land/sea boundary. Wait's solution has been used to compute ground loss for frequencies between 20 and 200 kHz.[7]

Earth curvature should be taken into account at l.f. because sea gain can be influenced by land/sea boundaries whose distance from an aerial may be a significant fraction of the Earth's circumference. Earth curvature also has some effect on sea gain at m.f. In planning broadcasting services, interest is confined to the dependence of sea gain on the distance between an aerial and a coastline, for values of ψ between 1° and 15°. Unfortunately the curves contained in Reference 7 do not cover the range of angles and distances of interest and the theory contained in Reference 6 is difficult to apply. In the simpler approach described here, Andersen's flat-Earth theory has been modified to take Earth curvature into account.

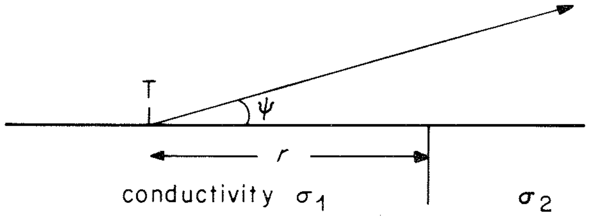

Fig. 2 - An aerial near a discontinuity in ground conductivity

Fig. 2 shows a short vertical aerial (T) near a discontinuity in ground conductivity. At l.f. and m.f., both land and sea may be regarded as pure conductors rather than as lossy dielectrics. With this assumption, Equation (2) of Andersen's paper simplifies to†

$$A = \frac{1+R_{\mathrm{v}}{}^{(1)}}{2}\cos\psi - \mathrm{j}\frac{\sqrt{\sigma_1}-\sqrt{\sigma_2}}{\sqrt{\sigma_2}}\cdot\frac{1+R_{\mathrm{v}}{}^{(2)}}{2}\,\frac{1}{(t/p)-\mathrm{j}}\left\{\sqrt{\frac{2t}{p}}\left[\frac{1}{2}-\mathrm{j}\frac{1}{2}-F(t)\right]-\mathrm{e}^{-\mathrm{j}t}w^*\right\} \qquad (2)$$

where A = vertical radiation pattern of aerial

ψ = angle of elevation of sky wave at aerial

σ_1 = ground conductivity at aerial

σ_2 = ground conductivity beyond discontinuity

$R_{\mathrm{v}}{}^{(1)}$ = Fresnel reflection coefficient for ground at aerial

$R_{\mathrm{v}}{}^{(2)}$ = Fresnel reflection coefficient for ground beyond discontinuity

$p = \dfrac{kr}{2x_1}$ (numerical distance to discontinuity)

$t = kr\,(1-\cos\psi)$

r = distance from terminal to boundary, measured in direction of propagation

$x_1 = 18\sigma_1/f$

f = frequency

$k = 2\pi/\lambda$

λ = wavelength

$F(t) = \displaystyle\int_0^t \frac{\mathrm{e}^{-\mathrm{j}x}}{\sqrt{2\pi x}}\,\mathrm{d}x = C - \mathrm{j}S$ (the complex Fresnel integral)

$w = \mathrm{e}^{-p}\,\mathrm{erfc}\,(-\mathrm{j}\sqrt{p})$

The asterisk denotes the complex conjugate of w.

The first term in Equation (2) is the v.r.p. of an aerial situated on a homogenous Earth of conductivity σ_1

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† In Equation (2) the time factor $\exp(\mathrm{j}\omega t)$ has been omitted. Andersen follows Norton[8] in using the $\exp(-\mathrm{j}\omega t)$ time factor but the opposite sign convention, used here, is now generally accepted.

and the remaining terms may be regarded as corrections accounting for the change in conductivity at the discontinuity. It can be shown that the discontinuity behaves as a secondary source excited by the ground wave when r is large.

In Equation (2) the distance to the far aerial is assumed to be large compared with r. The phase of the contribution from the discontinuity will be modified if this conditions is not satisfied; in general the modification will depend on the number of hops in the ionospheric propagation path. The phase of the contribution will be further modified by changes in path difference resulting from curved-Earth geometry and it can be shown that the modification caused by Earth curvature is much greater than that caused by the finite distance to the far aerial.

The contribution from the discontinuity will also be modified if Earth curvature affects the amplitude and phase of the ground wave arriving at the discontinuity. Comparison of flat-Earth and curved-Earth propagation curves shows that the additional ground-wave attenuation caused by Earth curvature is less than 6 dB for distances up to 400 km at l.f. and 200 km at m.f. As the additional phase retardation caused by imperfect ground conductivity never exceeds 180° with a flat Earth,[8] it is reasonable to assume that the phase of the ground wave is not greatly affected by Earth curvature within the range of distances mentioned above. Provided r lies within this range, therefore, Earth curvature may be taken into account simply by replacing the phase retardation t in Equation (2) by the retardation t' which applies when the Earth is curved, and by replacing ψ by the angle ψ' shown in Fig. 3 when evaluating $R_v^{(2)}$. Formulae for calculating t' and ψ', both of which depend on the number of ionospheric hops, are derived in the Appendix.

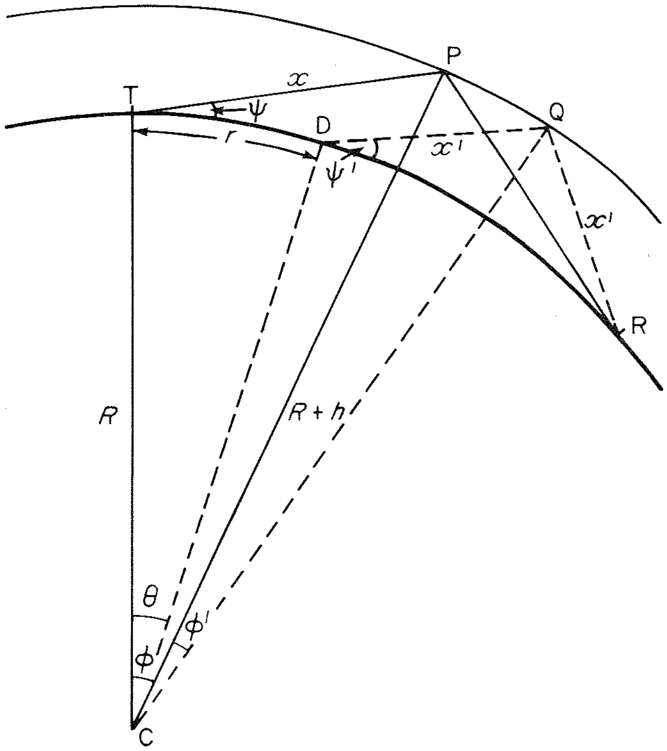

Fig. 3 - Curved-Earth geometry

The procedure described above can also be justified by considering the derivation of Equation (2), which made use of the Compensation Theorem.[9] In this method a distant transmitting aerial is assumed to illuminate the surface; the scalar product of the magnetic field due to the distant aerial and that due to the aerial T is then integrated over the part of the surface beyond the discontinuity (i.e. the surface whose conductivity is σ_2). The greatest contribution to the integral comes from the region nearest the discontinuity. If the Earth is curved the phase of this contribution is retarded by the modified value of t and its amplitude depends on ψ', the angle of incidence of the radiation from the distant aerial, rather than on ψ.

The modification to Equation (2) described above makes no allowance for diffraction around the curvature of the Earth. It is therefore valid only for values of ψ greater than 3° and for values of r up to 400 km at l.f. and 200 km at m.f. Wait's theory[6] may be used for lower radiation angles and for greater distances but should be applied with caution because it assumes that the distant aerial is at infinity.

If the Earth is assumed initially to consist entirely of land of uniform conductivity, sea gain will occur if the land on either side of the discontinuity is replaced by sea water. The sea gain in then given in decibels by

$$G = 20 \log_{10} \left| \frac{2A}{(1 + R_{vL}) \cos \psi} \right| \qquad (3)$$

where R_{vL} denotes the reflection coefficient for the land, at elevation angle ψ, and A is calculated by Equation (2), assuming a land/sea boundary. The sea may lie on either side of the boundary.

When the aerial is situated inland from the coast it is the section beyond the boundary which consists of sea water. Fig. 4, calculated by the method described here, shows how sea gain depends on the distance between the aerial and the coastline. When the aerial is on the coast the sea gain has the value which would apply if it were entirely surrounded by sea water but as it moves inland the sea gain decreases. The oscillations in the curves are the result of interference between direct radiation from T and re-radiation from the boundary; they decay rapidly at 1 MHz because ground-wave attenuation reduces the strength of the re-radiated wave. Fig. 4 shows that the effect of Earth curvature is greatest for one-hop propagation and that the variation of sea gain with distance approaches the flat-Earth curve as the number of hops increases.

If the aerial is situated on the coast and the sea is of limited extent, the first section of ground may be assumed to consist of sea water. Fig. 5 shows how the theoretical sea gain varies as the distance between the aerial and the land/sea boundary increases. When this distance is zero, land extends in the direction of propagation and there is no sea gain. As the distance increases the sea gain oscillates

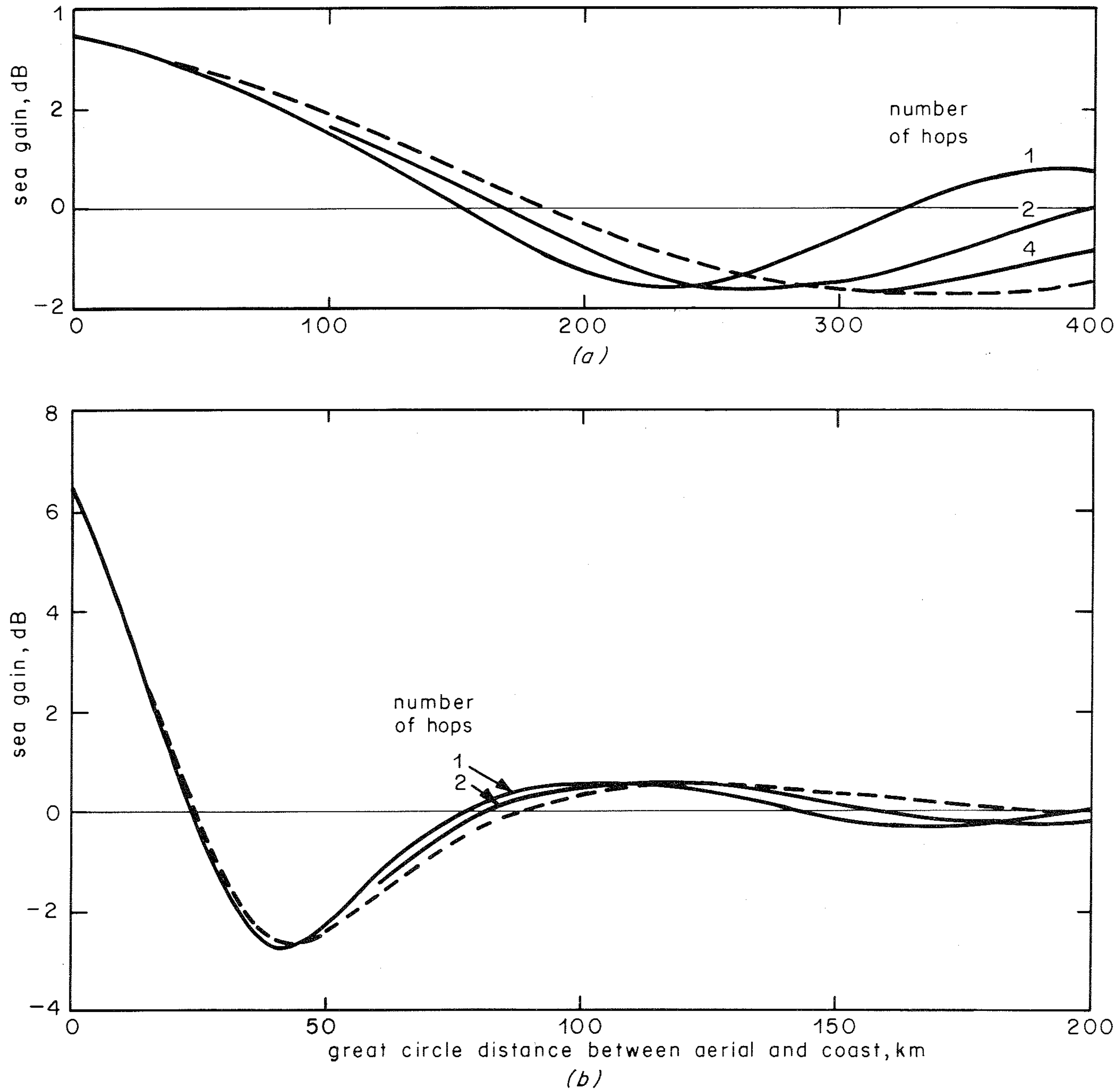

Fig. 4 - Sea gain for an inland aerial

(*a*) 200 kHz (*b*) 1 MHz

——— curved-Earth - - - - - flat-Earth

Radiation angle 3° Land conductivity 10 mS/m

about the value which would apply if the sea were of infinite extent. The oscillations decay less rapidly in this case because the ground wave propagates over sea water and suffers less attenuation. As before, the effect of Earth curvature is greatest for one-hop propagation.

4. Discussion and conclusions

The curves of Fig. 1 provide an accurate and convenient way of estimating the sea gain which occurs when an aerial is situated on an open coastline rather than well inland. They have been used to calculate the effective sea gain which occurs when one or more propagation modes are present. Curves showing the resulting variation of sea gain with path length are contained in Fig. 1 of a companion report[10] and have been adopted by the CCIR.[11] These curves take all Earth curvature effects, including diffraction, fully into account.

Although the method described in this report for calculating sea gain in the presence of a land/sea boundary

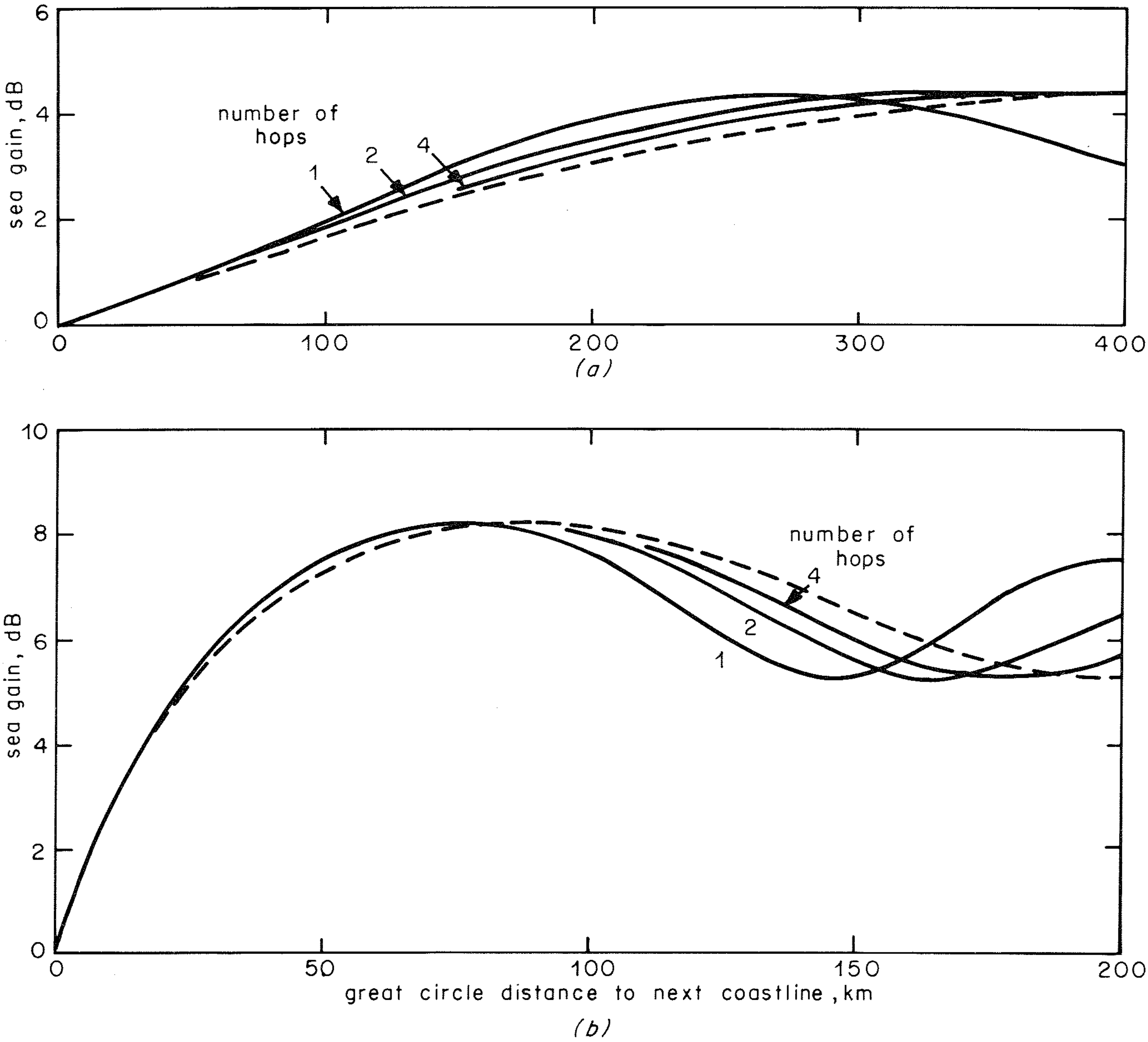

Fig. 5 - Sea gain for an aerial on the coast

(*a*) 200 kHz (*b*) 1 MHz

———— curved-Earth - - - - - - - flat-Earth

Radiation angle 3° Land conductivity 10 mS/m

takes Earth curvature into account, it neglects diffraction. Consequently the method overestimates sea gain for radiation angles less than 3°, especially at l.f. where the error may exceed 4 dB at angles less than 1°. This difficulty is partly overcome in the approximate method for estimating sea gain which is described in the companion report.[10] In the approximate method the sea gain G is assumed to vary linearly between zero and a maximum value G_0 as the distance between the aerial and the land/sea boundary increases. The value of G_0 is derived from Fig. 1 and therefore takes diffraction into account, but the method described in Section 3 of this report, which neglects diffraction, is used to determine the distance r_1 at which G first falls to zero when an aerial moves inland (Fig. 4) or the distance r_2 at which G first reaches its asymptotic limit when an aerial is situated on the coast (Fig. 5), assuming single-hop propagation in each case. Figs. 4 and 5 show that Earth curvature does not greatly affect these two distances and consequently no great error would be introduced into the calculation of r_1 and r_2 if Earth curvature were neglected altogether.

The effects of Earth curvature on sea gain may be summarised as follows:

(1) If an aerial is situated on the coast, sea gain for low-angle propagation modes may be several decibels less than the value which would be calculated if Earth curvature were disregarded. (Fig. 7 of CCIR Report 575, reproduced in Reference 10, takes Earth curvature into account.)

(2) If the separation between an aerial and a land/sea boundary increases, the separation at which sea gain

first attains the value which would apply if the Earth were homogeneous is not greatly affected by Earth curvature.

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6. Appendix:

Formulae for Curved-Earth Geometry

Fig. 3 shows a single-hop path TPR together with the path TDQR followed by a wave which propagates via a discontinuity at D. To use the modified form of Equation (2), both the path difference TDQR—TPR and the angle ψ' must be calculated.

The direct path TPR is equal to $2x$, which may be calculated by applying the cosine rule to the triangle TPC and solving the quadratic equation which results. The solution is

$$2x = -b + (b^2 + 4c)^{1/2} \tag{4}$$

where $b = 2R \sin \psi$
$c = (R + h)^2 - R^2$
R = radius of the Earth
h = height of reflecting layer

The true Earth radius (6371 km) is used for the calculation because sky-waves radiated at the angles of elevation of interest pass rapidly out of the Earth's atmosphere and are not greatly influenced by atmospheric refraction. The height of the reflecting layer is assumed to be 90 km because E-layer propagation always applies to low-angle propagation at l.f. and m.f.

Application of the sine rule to the same triangle then gives the angle ϕ subtended by each half of the direct path as follows

$$\sin\phi = \frac{x \cos \psi}{R + h} \tag{5}$$

The corresponding angle ϕ' subtended by each half of the indirect path DQR is then given by

$$\phi' = \phi - \frac{\theta}{2} \tag{6}$$

where θ is the angle (equal to r/R) subtended by the arc TD. Knowing ϕ', the indirect ionospheric path length $2x'$ may be calculated from the triangle CDQ as follows

$$2x' = 2[R^2 + (R + h)^2 - 2R(R + h) \cos \phi']^{1/2} \tag{7}$$

Since the total indirect path length is $r + 2x'$, the path difference is equal to $r + 2x' - 2x$. Multiplication by $2\pi/\lambda$ then gives the path difference t' in radians which is substituted in Equation (2). Finally the angle ψ' is calculated by applying the sine rule to the triangle CDQ, as follows:

$$\cos \psi' = \frac{(R + h) \sin \phi'}{x'} \qquad (8)$$

For two-hop propagation both the path difference and the angle ψ' would be unchanged if the arc TD were divided equally between the two ends of the path in order to simplify the geometry. If r were replaced by $r/2$, Fig. 3 would then represent one half of the complete path and x' and ψ' could be calculated from the formulae given above by halving θ. Since the direct and indirect path lengths for two-hop propagation are $4x$ and $r + 4x'$ respectively, the path difference is given by $r + 4(x' - x)$. This method can be extended to n-hop propagation by replacing θ by θ/n in Equation (6), the path difference then being $r + 2n(x' - x)$.

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